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MA204 - Linear Algebra and Matrices
Problem Sheet - 1

Matrices and Gaussian Elimination

1. Sketch these three lines and decide if the equations are solvable:

$$\textbf{3 by 2 system} \qquad \begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 1. \end{aligned}$$

   What happens if all right-hand sides are zero? Is there any nonzero choice of right-hand side that allows the three lines to intersect at the same point?

2. (Recommended) Describe the intersection of the three planes $u + v + w + z = 6$ and $u + w + z = 4$ and $u + w = 2$ (all in four-dimensional space). Is it a line or a point or an empty set? What is the intersection if the fourth plane $u = -1$ is included? Find a fourth equation that leaves us with no solution.

3. Draw the two pictures in two planes for the equations $x - 2y = 0$, $x + y = 6$.

4. When equation 1 is added to equation 2, which of these are changed: the planes in the row picture, the column picture, the coefficient matrix, the solution?

5. If (a, b) is a multiple of (c, d) with $abcd \neq 0$, show that (a, c) is a multiple of (b, d). This is surprisingly important: call it a challenge equation. You could use numbers first to see how a, b, c, and d are related. The question will lead to:

   If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has dependent rows, then it has dependent columns.

$$\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 3 \\ 2x + 3y + 2z &= 5. \end{aligned}$$

   The first two planes meet along a line. The third plane contains that line, because if x, y, z satisfy the first two equations then they also ____. The equations have infinitely many solutions (the whole line **L**). Find three solutions.

6. Normally 4 "planes" in four-dimensional space meet at a ____. Normally 4 column vectors in four-dimensional space can combine to produce b. what combination of $(1, 0, 0, 0)$, $(1, 1, 0, 0)$, $(1, 1, 1, 0)$, $(1, 1, 1, 1)$ produces $b = (3, 3, 3, 2)$? What 4 equations for x, y, z, t are you solving?

7. Choose a right-hand side which gives no solution and another right-hand side which gives infinitely many solutions. What are two of those solutions?

$$\begin{aligned} 3x + 2y &= 10 \\ 6x + 4y &= \underline{\quad}. \end{aligned}$$

8. For which numbers a does elimination break down (a) permanently, and (b) temporarily?

$$\begin{aligned} ax + 3y &= -3 \\ 4x + 6y &= 6. \end{aligned}$$

   Solve for x and y after fixing the second breakdown by a row exchange.

9. Which number d forces a row exchange, and what is the triangular system (not singular) for that d? Which d makes this system singular (no third pivot)?

$$\begin{aligned} 2x + 5y + z &= 0 \\ 4x + dy + z &= 2 \\ y - z &= 3. \end{aligned}$$

10. If rows 1 and 2 are the same, how far can you get with elimination (allowing row exchange)? If columns 1 and 2 are the same, which pivot is missing?

$$\begin{aligned} 2x - y + z &= 0 \\ 2x - y + z &= 0 \\ 4x + y + z &= 2 \end{aligned} \qquad\qquad \begin{aligned} 2x + 2y + z &= 0 \\ 4x + 4y + z &= 0 \\ 6x + 6y + z &= 2. \end{aligned}$$

11. (a) Construct a 3 by 3 system that needs two row exchanges to reach a triangular form and a solution.
    (b) Construct a 3 by 3 system that needs a row exchange to keep going, but breaks down later.

12. Three planes can fail to have an intersection point, when no two planes are parallel. The system is singular if row 3 of A is a ______ of the first two rows. Find a third equation that can't be solved if $x + y + z = 0$ and $x - 2y - z = 1$.

13. True or false:
    (a) If the third equation starts with a zero coefficient (it begins with $0u$) then no multiple of equation 1 will be subtracted from equation 3.
    (b) If the third equation has zero as its second coefficient (it contains $0v$) then no multiple of equation 2 will be subtracted from equation 3.
    (c) If the third equation contains $0u$ and $0v$, then no multiple of equation 1 or equation 2 will be subtracted from equation 3.

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